

Combined features in the primordial spectra induced by a sudden turn in two-field DBI inflation

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Abstract

We investigate the features generated by a sharp turn along the inflationary trajectory in a two-field model of Dirac-Born-Infeld inflation, where one of the fields is heavy. Distinct features are generated by two different effects: the mixing of the light and heavy modes during the turn, on the one hand, and the resonance between the oscillations along the heavy direction after the turn, on the other hand. Contrary to models with standard kinetic terms, the resonance effect is not strongly suppressed because the action contains derivative interactions. Working in the potential basis, we study the oscillations after the turn and compute the amplitude of the mixing and resonance features in the power spectrum, as well as in the bispectrum for the latter effect. We find that the amplitudes and positions of these combined features obey specific consistency relations, which could be confronted with cosmological data.

PACS numbers:

I. INTRODUCTION

The detailed analysis of primordial density fluctuations represents the main window onto high energy physics at work during cosmic inflation. In the inflationary scenario, the primordial fluctuations are generated from the quantum fluctuations of one (or several) scalar field(s), called the inflaton(s), characterized by a light mass, i.e. smaller than the Hubble parameter, during inflation (see e.g. [1] for a pedagogical introduction). Primordial fluctuations are observable today via their imprint on the cosmic microwave background (CMB) and large scale structures.

From the perspective of inflationary model building, light scalar fields are usually not the only scalar fields present. In general, one also finds scalar fields with masses of the order of the Hubble parameter, or even much heavier. However, in contrast with inflatons (i.e. light modes), these heavy fields (heavy modes) are usually considered as irrelevant from an observational point of view as their fluctuations are significantly suppressed on cosmological scales.

Recently, however, it was pointed out that such heavy modes can in fact affect the fluctuations generated during inflation when the inflationary trajectory in field space is bent. Indeed, in such situations, the heavy field can be displaced from the minimum of the potential as a consequence of the centrifugal force induced by the turn of the trajectory. For a moderate bending of the trajectory, it was shown that the system is described as an effective single light field model with a reduced speed of sound [2–4] (see also [5–23] for other works investigating the effects of heavy modes in terms of an effective theory for the light field). Moreover, for a sufficiently sharp bending of the trajectory, the heavy field oscillates around its minimum after the turn and the effective single light field description is no longer valid.

A sharp turn can induce two types of features, which are potentially observable in the primordial spectra such as the power spectrum and bispectrum. The first type of features is induced around the scale that crosses the Hubble scale at the time of the turn through a large mixing between the light and heavy modes during the turn [24–27]. In addition to this, when the turn is sufficiently sharp to induce oscillations of the heavy field, another type of features is generated. The latter appear around the scale that crosses the mass scale of the heavy field at the time of the turn through the resonance between the oscillations in the background trajectory and the inflaton fluctuations [28–31]. Although it was shown in Refs [32, 33] that this resonance effect is not efficient for canonical scalar fields, the features can nevertheless be large when the light and heavy modes are coupled via derivative interactions [31] (see also [34]).

Detection of a combination of both types of features could provide compelling evidence for the existence of such heavy modes during inflation. Indeed, with only a single type of features analyzed, it is difficult to obtain a large statistical significance because one cannot predict the position of the feature, which depends on when the turn occurred during inflation. However, it is possible to increase the significance of the signal if different and correlated features can be detected in the power spectrum and bispectrum [35–38]. This is the case here, as the relative position between mixing and resonance features is determined by the ratio between the heavy mass and the Hubble parameter.

The purpose of this paper is to investigate how much the heavy field can be excited during a sharp turn in a model with derivative interactions, so that both mixing and resonance features are expected. A model of inflation that naturally provides such type of interactions

is based on the Dirac-Born-Infeld (DBI) Lagrangian with multiple fields [39–41]. Whereas derivative interactions were treated perturbatively in the previous work [31], DBI inflation automatically includes higher-order terms, which could become important at the turn. In this framework, the goal of this work is to determine the efficiency of the heavy mode excitation, depending on the model parameters, and to establish a relation between the features due to the mixing and resonance effects.

The organization of this paper is as follows. In Section II, we introduce our two-field model and derive the background equations of motion, which are reexpressed in a convenient basis. In Section III, we concentrate on the evolution of the trajectory just after the turn, in particular on its oscillations due to the excitation of the heavy mode. After deriving analytical results, we study numerically a specific example. Section IV is devoted to the features generated in the power spectrum by the mixing effect and the resonance effect. Features in the bispectrum are also discussed. We summarize our results in the final section. In the appendix, we give some details about the resonance feature in the power spectrum.

II. BACKGROUND EVOLUTIONS

In this section, we introduce the two-field model of DBI inflation that we will study, and derive the relevant background equations of motion.

A. Two-field DBI inflation

Following the previous works [39–41], we consider a model with two scalar fields ϕ^I ($I = 1, 2$), governed by the action

$$S = \int d^4x \sqrt{-g} P(X^{IJ}, \phi^I), \quad (1)$$

where P is a function of the scalar fields and of their kinetic terms

$$X^{IJ} \equiv -\frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J, \quad (2)$$

and g is the determinant of the spacetime metric $g_{\mu\nu}$.

For two-field DBI inflation models, $P(X^{IJ}, \phi^I)$ is explicitly given by

$$P(X^{IJ}, \phi^I) = \tilde{P}(\tilde{X}, \phi^I) = -\frac{1}{f(\phi^I)} \left(\sqrt{1 - 2f(\phi^I)\tilde{X}} - 1 \right) - V(\phi^I), \quad (3)$$

where $f(\phi^I)$ and $V(\phi^I)$ are functions of the scalar fields. Here, \tilde{X} is defined in terms of the determinant (we use Einstein's implicit summation rule for the scalar field indices)

$$\begin{aligned} \mathcal{D} &= \det(\delta_J^I - 2fX_J^I) \\ &= 1 - 2fG_{IJ}X^{IJ} + 4f^2X_I^{[I}X_J^{J]}, \end{aligned} \quad (4)$$

as

$$\tilde{X} = \frac{(1 - \mathcal{D})}{2f}, \quad (5)$$

where G_{IJ} is the metric in the field space. In the context of string theory, the DBI action describes the effective dynamics of a D3 brane in a higher-dimensional background spacetime and $f(\phi^I)$ is related to the warp factor $h(\phi^I)$ of this higher-dimensional spacetime and the brane tension T_3 as

$$f(\phi^I) \equiv \frac{h(\phi^I)}{T_3}. \quad (6)$$

Note that the action (1) contains derivative interactions such as $(X^{12})^2$ and $X^{11}X^{22}$, which were considered in Ref. [31], as well as self couplings and higher-order terms. In the DBI inflation model, the magnitude of all these couplings is determined by the single function $f(\phi^I)$, or equivalently by the effective sound speed c_s defined below. A small speed of sound corresponds to large derivative interactions.

B. Background equations of motion

In a spatially homogeneous and isotropic spacetime, endowed with the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (7)$$

the evolution of the scale factor $a(t)$ is governed by the Friedmann equations

$$H^2 = \frac{1}{3} \left(\frac{1}{(1+c_s)c_s} G_{IJ} \dot{\phi}^I \dot{\phi}^J + V \right), \quad \dot{H} = -\frac{1}{2c_s} G_{IJ} \dot{\phi}^I \dot{\phi}^J \equiv -H^2 \epsilon, \quad (8)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and a dot denotes a derivative with respect to the cosmic time t . We use units such that $M_P \equiv (8\pi G)^{-1/2} = 1$. Here, c_s is the effective sound speed, corresponding to the propagation speed of the perturbations, and is given by [39]

$$c_s \equiv \sqrt{\frac{\tilde{P}_{,\tilde{X}}}{\tilde{P}_{,\tilde{X}} + 2\tilde{X}\tilde{P}_{,\tilde{X}\tilde{X}}}} = \sqrt{1 - 2f\tilde{X}}, \quad (9)$$

where $_{,\tilde{X}}$ means the partial derivative with respect to \tilde{X} . Note that \tilde{X} coincides with $X \equiv G_{IJ}X^{IJ}$ in the homogeneous background because all spatial derivatives vanish. From the action (3), we can show that

$$\tilde{P}_{,\tilde{X}} = \frac{1}{c_s}. \quad (10)$$

By introducing the components of the acceleration in curved space (here the field space), which we can write as

$$\mathcal{D}_t \dot{\phi}^I \equiv \ddot{\phi}^I + \Gamma_{JK}^I \dot{\phi}^J \dot{\phi}^K, \quad (11)$$

and the simplified notation $_{,J} \equiv \partial/\partial\phi^J$, the equations of motion for the homogeneous scalar fields are

$$\mathcal{D}_t \dot{\phi}^I + (3 - \epsilon_s) H \dot{\phi}^I - c_s G^{IJ} \tilde{P}_{,J} = 0; \quad \tilde{P}_{,J} = -V_{,J} + \frac{(1 - c_s)^2}{2c_s} \frac{f_{,J}}{f^2}, \quad \epsilon_s \equiv \frac{\dot{c}_s}{H c_s}, \quad (12)$$

or, in an even more compact form,

$$a^{-3} \mathcal{D}_t \left(a^3 \frac{1}{c_s} \dot{\phi}^I \right) = \tilde{P}_{,I}, \quad (13)$$

where we have used the field space metric G_{IJ} to lower the field index I , so that $\dot{\phi}_I \equiv G_{IJ}\dot{\phi}^J$. \mathcal{D}_t acts as an ordinary time derivative on field space scalars (i.e. quantities without field space indices) and $\mathcal{D}_t G_{IJ} = 0$.

Since our purpose is to focus on the effects of a reduced sound speed during the turn and on the resonance associated with the background oscillation after it, from now on, we consider a simplified model characterized by $G_{IJ} = \delta_{IJ}$ and $f = \text{const.}$ This implies $\tilde{P}_{,I} = -V_{,I}$, which simplifies Eq. (12) as

$$\ddot{\phi}^I + (3 - \epsilon_s)H\dot{\phi}^I + c_s V^{,I} = 0. \quad (14)$$

From this equation, one sees that the sound speed, which is always smaller than 1, has two consequences: a modification of the Hubble friction and a flattening of the effective potential felt by the scalar fields.

C. Kinematic and potential bases

We will now solve the evolution equation (14) following Ref. [26], including the new effects due to the nontrivial sound speed. First of all, let us define the light and heavy directions in field space, corresponding to the basis e_m^I that diagonalizes the Hessian matrix of the potential, i.e. such that

$$V_{,mn} \equiv e_m^I e_n^J V_{,IJ} = \text{diag}\{m_l^2, m_h^2\}. \quad (15)$$

Because of the flattening of the potential due to the sound speed, the masses are effectively reduced by a factor $\sqrt{c_s}$. We use the terms “light” and “heavy” for these effective masses and assume the hierarchy

$$\sqrt{c_s}m_l \ll H \ll \sqrt{c_s}m_h. \quad (16)$$

Following the terminology introduced in Ref. [26], we will call the basis that diagonalizes the Hessian matrix of the potential, the “potential basis”. In a similar fashion, we call the basis associated with the usual adiabatic-entropic decomposition, the “kinematic basis”.

To describe the deviation between the velocity (adiabatic) direction

$$n^I = \frac{\dot{\phi}^I}{\dot{\sigma}}, \quad \dot{\sigma} \equiv \sqrt{\delta_{IJ}\dot{\phi}^I\dot{\phi}^J}, \quad (17)$$

and the light direction, we consider the evolution of the angle ψ between the adiabatic and light directions. The components of the adiabatic unit vector in the potential basis can be explicitly expressed in terms of ψ as

$$\{n_1, n_2\} = \{\cos \psi, \sin \psi\}. \quad (18)$$

The angle ψ can be written as the difference

$$\psi = \theta_k - \theta_p, \quad (19)$$

where θ_k and θ_p are, respectively, the angles of the kinematic and potential bases with respect to the original field basis. While the angle θ_p can be determined by the shape of the

potential, we should solve Eq. (14) to know the evolution of the other angle θ_k . Moving to the kinematic basis, the equations of motion (14) can be rewritten as

$$(3 + \epsilon_\sigma - \epsilon_s)H\dot{\sigma} + c_s V_{,\sigma} = 0, \quad V_{,\sigma} \equiv n^I V_{,I}, \quad (20)$$

$$\dot{\sigma}\dot{\theta}_k + c_s V_{,s} = 0, \quad V_{,s} \equiv s^I V_{,I}, \quad \epsilon_\sigma \equiv \frac{\ddot{\sigma}}{H\dot{\sigma}}. \quad (21)$$

Combining these equations yields the evolution equation for θ_k ,

$$\ddot{\theta}_k + (3 + 2\epsilon_\sigma - 2\epsilon_s)H\dot{\theta}_k + c_s V_{,\sigma s} = 0. \quad (22)$$

Hence, substituting the expression (19) into Eq. (22), we obtain

$$\ddot{\psi} + 3H \left(1 + \frac{2}{3}(\epsilon_\sigma - \epsilon_s)\right) \dot{\psi} + \frac{c_s}{2} (m_h^2 - m_l^2) \sin(2\psi) = -\ddot{\theta}_p - 3H \left(1 + \frac{2}{3}(\epsilon_\sigma - \epsilon_s)\right) \dot{\theta}_p. \quad (23)$$

In order to obtain simple analytical solutions, we will neglect $\epsilon_\sigma - \epsilon_s$ in Eq. (23). Because the velocity contains a contribution from the heavy field $\dot{\phi}_h$, its derivative induces a large factor m_h . Evaluating these contributions, we find

$$\epsilon_\sigma - \epsilon_s \sim \frac{\sqrt{c_s} m_h}{H} \left(\frac{\dot{\phi}_h}{\dot{\sigma}} \right)^2, \quad (24)$$

using $(\dot{\phi}_h/c_s) \sim m_h^2 \phi_h$ and $\dot{\phi}_h \sim \sqrt{c_s} m_h \phi_h$, which can be shown to be consistent with the evolution equation (14) for the heavy scalar field with $\sqrt{c_s} m_h \gg H$. As will be shown in Sec. IV, this is the same order as the amplitude of the features in the power spectrum induced by the sharp turn. Therefore, $\epsilon_\sigma - \epsilon_s$ can be safely neglected for a reasonable modulation of the power spectrum. We also assume that the angle ψ is sufficiently small so that the sine can be replaced by its argument. With these approximations and by neglecting m_l^2 , the evolution equation (23) reduces to

$$\ddot{\psi} + 3H\dot{\psi} + c_s m_h^2 \psi = -\ddot{\theta}_p - 3H\dot{\theta}_p. \quad (25)$$

It is worth mentioning that when the condition for inflation

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\sigma}^2}{c_s H^2} \ll 1, \quad (26)$$

as well as $\epsilon_\sigma - \epsilon_s \ll 1$ are satisfied, the sound speed c_s is approximately determined by the potential as

$$c_s \simeq \sqrt{\frac{1}{1 + 2\epsilon_V f H^2}} \equiv c_{s0}, \quad \epsilon_V \equiv \frac{1}{2} \left(\frac{V_{,\sigma}}{V} \right)^2, \quad (27)$$

and $\epsilon \simeq c_s \epsilon_V$.

III. EXCITATION OF THE HEAVY MODE

A. Efficiency of the excitation

Assuming that H and $\sqrt{c_s}m_h$ remain approximately constant during the turn, we can formally solve Eq. (25) as

$$\psi(t) = - \int^t dt' G(t, t') \left[\ddot{\theta}_p(t') + 3H\dot{\theta}_p(t') \right], \quad (28)$$

using the retarded Green's function given by

$$G(t, t') = \Theta(t - t') \frac{\sin(\omega(t - t'))}{\omega} e^{-\frac{3}{2}H(t-t')}, \quad \omega = \sqrt{c_s m_h^2 - \frac{9}{4}H^2}, \quad (29)$$

where Θ is the Heaviside distribution. Because of the assumption (16), the frequency ω can be approximated by

$$\omega \simeq \sqrt{c_s} m_h. \quad (30)$$

The precise evolution of the angle $\theta_p(t)$ depends on the details of the potential. In order to work with analytical expressions, here, we simply characterize the turn by two parameters, as in [26], and use the function

$$\dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2}, \quad (31)$$

where the turn is assumed to occur at time $t = 0$. $\Delta\theta$ represents the global variation of the angle during the turn and μ is related to the duration of the turn by $\Delta t_{\text{turn}} \sim \mu^{-1}$. When the slow-roll conditions are satisfied, μ can be estimated as

$$\mu \sim s\dot{\sigma} \sim s c_s \sqrt{\epsilon_V} H, \quad (32)$$

where s represents the “sharpness” of the turn in field space: $s \sim (\partial\theta_p/\partial\sigma)/\Delta\theta$.

Plugging the expression (31) into Eq. (28), we get

$$\psi(t) = -\frac{\Delta\theta}{2} \sqrt{1 + \frac{9H^2}{4\omega^2}} e^{-\frac{3}{2}Ht} \Re \left[e^{i\alpha + \varphi^2/2} e^{-i\omega t} \text{erfc} \left(-\frac{\mu t - \varphi}{\sqrt{2}} \right) \right], \quad (33)$$

where \Re denotes the real part of the argument, $\text{erfc}(z) \equiv 1 - \text{erf}(z)$ is the complementary error function and we have introduced the parameters

$$\alpha := \arctan \left(\frac{3H}{2\omega} \right), \quad \varphi := \frac{\omega}{\mu} \sqrt{1 + \frac{9H^2}{4\omega^2}} e^{i(\frac{\pi}{2} - \alpha)}. \quad (34)$$

In the following, we will be mainly interested in sharp turns, corresponding to $\mu/\omega \gtrsim 1$, which lead to post-turn oscillations. In this case, the expression (33) is approximated by

$$\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{\omega^2}{2\mu^2}} \text{erfc} \left(-\frac{\mu t}{\sqrt{2}} \right) e^{-\frac{3}{2}Ht} \cos \left(\omega t - \alpha - \frac{3H\omega}{2\mu^2} \right). \quad (35)$$

After the turn, i.e. for $\mu t \gtrsim 1$, ψ starts to oscillate around the light direction, with the damping factor $e^{-\frac{3}{2}Ht}$. In terms of ψ , the evolution of the heavy field is given by

$$\dot{\phi}_h = \dot{\sigma} \sin \psi \simeq \dot{\sigma} \psi, \quad (36)$$

which expresses the oscillations of the heavy scalar field excited by the turn.

Since it is reasonable to consider that the energy of the excited oscillations is mainly extracted from the kinetic energy before the turn, we define the efficiency of the excitation through

$$\xi_{\text{osc}} \equiv \left(\frac{\dot{\phi}_{h,\text{max}}}{\dot{\sigma}} \right)^2 \simeq (\psi_{\text{max}})^2, \quad (37)$$

where we have assumed in the second equality that the velocity $\dot{\sigma}$ is approximately constant during the turn (which will be verified in the explicit example we consider later). Using the expression (35), we find that ξ_{osc} is given by

$$\xi_{\text{osc}} \simeq e^{-\frac{\omega^2}{\mu^2}} (\Delta\theta)^2. \quad (38)$$

In general, the prefactor $e^{-\omega^2/\mu^2}$ changes if we use a time dependence for $\dot{\theta}_p$ that differs from Eq. (31). However, in the limit $\mu \gg \omega$, the efficiency reaches its maximum value

$$\xi_{\text{osc,max}} \simeq (\Delta\theta)^2, \quad (39)$$

which only depends on the global variation of the angle, $\Delta\theta$.

Using the expressions (30) and (32), the condition $\mu \gg \omega$ can be expressed as

$$\frac{m_h}{H} \ll s\sqrt{c_s\epsilon_V} \simeq s\sqrt{\epsilon}. \quad (40)$$

Thus, in terms of the slow-roll parameter ϵ , the sound speed does not appear in the above condition. However, for a given potential, a small sound speed makes the heavy mode more difficult to excite. This is because, though the heavy mass is suppressed by the small sound speed, the incident velocity, $\dot{\sigma}$, is reduced as well. By contrast, the maximum efficiency (39) does not depend on the sound speed but only on the global variation of the angle $\Delta\theta$.

For completeness, let us also briefly mention the analytic result in the soft turn case, even if it does not induce any resonance. In the limit $\mu \ll \omega$, the evolution of ψ , before and during the turn, is approximately given by

$$\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}} \frac{\mu^2}{\omega^2} e^{-\frac{1}{2}\mu^2 t^2} \left(\mu t - 3\frac{H}{\mu} \right). \quad (41)$$

Moreover, long *after* the turn, when $\mu t \gtrsim \omega/\mu \gg 1$, ψ behaves like

$$\psi(t) \approx -\Delta\theta \left[e^{-\frac{\omega^2}{2\mu^2}} e^{-\frac{3}{2}Ht} \cos \left(\omega t - \alpha - \frac{3H\omega}{2\mu^2} \right) - \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2} \frac{1}{\mu t} \right]. \quad (42)$$

This shows that, after the turn, the heavy mode oscillates around the minimum, but with a tiny amplitude since $\omega/\mu \gg 1$.

B. Numerical analysis

In the previous subsection, we have estimated analytically the efficiency of the excitation by assuming $\sqrt{c_s} m_h$ to be constant during the turn. In this subsection, we consider an explicit model and compute numerically the efficiency parameter ξ_{osc} , in order to compare the result with our analytical estimate (39).

1. Potential

As an example, we consider the same potential as the one used in Ref. [26], even if the kinetic terms are now non canonical. In terms of the two scalar fields ϕ and χ , the potential is given by

$$V(\phi, \chi) = \frac{1}{2} M^2 \cos^2 \left(\frac{\Delta\theta}{2} \right) [\chi - (\phi - \phi_0) \tan \Xi]^2 + \frac{1}{2} m_\phi^2 \phi^2, \quad (43)$$

where the function $\Xi(\phi)$ is defined by

$$\Xi(\phi) = \frac{\Delta\theta}{\pi} \arctan[s(\phi - \phi_0)]. \quad (44)$$

The quantity s is a constant parameter that controls the sharpness of the turn and is related to μ by

$$\mu \simeq \sqrt{2} \dot{\phi} s \simeq \frac{2}{\sqrt{3}} c_s m_\phi s. \quad (45)$$

The mass parameters m_ϕ and M are chosen so that the condition $\sqrt{c_s} m_\phi \ll H \ll \sqrt{c_s} M$ is satisfied.

The potential (43) has been constructed so that the heavy direction, which corresponds approximately to

$$\phi_h \simeq \chi - (\phi - \phi_0) \tan \Xi, \quad (46)$$

changes its direction in field space around $\phi = \phi_0$, $\chi = 0$.

2. Parameters

Our model contains six parameters. The turn itself is characterized by three parameters, $\{\Delta\theta, s, \phi_0\}$. The potential also depends on two mass parameters, m_ϕ and M , while the kinetic term depends on the parameter f . The Hubble scale H can be fixed by requiring that the observed power spectrum, $\mathcal{P}_{\zeta 0} \simeq 10^{-9}$ is approximately reproduced, for $\phi \simeq \phi_0$, in the limit $\Delta\theta \rightarrow 0$. This gives

$$H \simeq 2\pi M_P \sqrt{2c_s \epsilon \mathcal{P}_{\zeta 0}}, \quad (47)$$

where we have explicitly written the Planck scale M_P . Moreover, approximating $V \simeq m_\phi^2 \phi^2 / 2$, we find from $\epsilon \simeq c_s \epsilon_V$,

$$\phi_0 \simeq -M_P \sqrt{\frac{2c_s}{\epsilon}}, \quad (48)$$

and from Eq. (27),

$$f \simeq \frac{c_s}{2\epsilon M_P^2 H^2} \left(\frac{1}{c_s^2} - 1 \right). \quad (49)$$

Finally, using the Friedmann equation (8) within the slow-roll approximation, the light mass m_ϕ can be determined as

$$m_\phi \simeq H \sqrt{\frac{3\epsilon}{c_s}}. \quad (50)$$

To fix the parameters, we also assume the value of the slow-variation parameter $\epsilon (\simeq c_s \epsilon_V)$ to be 0.01. Given these constraints between the various parameters, it is more convenient to parametrize our system by the four parameters $\{\Delta\theta, s, M, c_s\}$.

3. Efficiency: numerical result

We have evaluated numerically the efficiency parameter (37) for various values of the parameters $\{\Delta\theta, s, M, c_s\}$ by solving the equations of motion (14). In Fig. 1, we have plotted our numerical estimate of the efficiency as a function of $\mu/\sqrt{c_s}M$, for various values of the angle $\Delta\theta$, of the ratio M/H and of the sound speed c_s . Comparing these results with the analytical expression (38), one finds that the latter provides a very good approximation provided the heavy mass is sufficiently large or the turn sufficiently sharp, i.e. $\mu \gg \sqrt{c_s}M$. In particular, one can check that the sharp-turn limit corresponds to $\Delta\theta^2$ and is indeed independent of the values of M and c_s .

The deviation from the analytical result when μ is not so large, can be explained by the dependence of the efficiency on the details of the evolution of the angle θ_p during the turn. Indeed, one does not expect the Gaussian approximation (31) to be accurate in this regime. Moreover, the assumption that $\sqrt{c_s}M$ is constant is also not valid during the turn.

In conclusion, while the efficiency depends on how the angle θ_p evolves during the turn when the turn is not very sharp, we have confirmed that in the sharp-turn limit, the efficiency only depends on the global variation of the angle, $\Delta\theta$.

IV. THE FEATURES IN THE PRIMORDIAL SPECTRA

In this section, we first discuss the features induced by the mixing effect in the power spectrum, then the features induced by the resonance effect both in the power spectrum and the bispectrum. Finally, we compare these two types of features and show that there exist simple relations between them, which depend only on parameters that are principle measurable.

A. Features induced by the mixing

Let us briefly summarize the analysis of [26] devoted to the features generated by the mixing caused by a sudden turn of the inflationary trajectory, for models with canonical kinetic terms

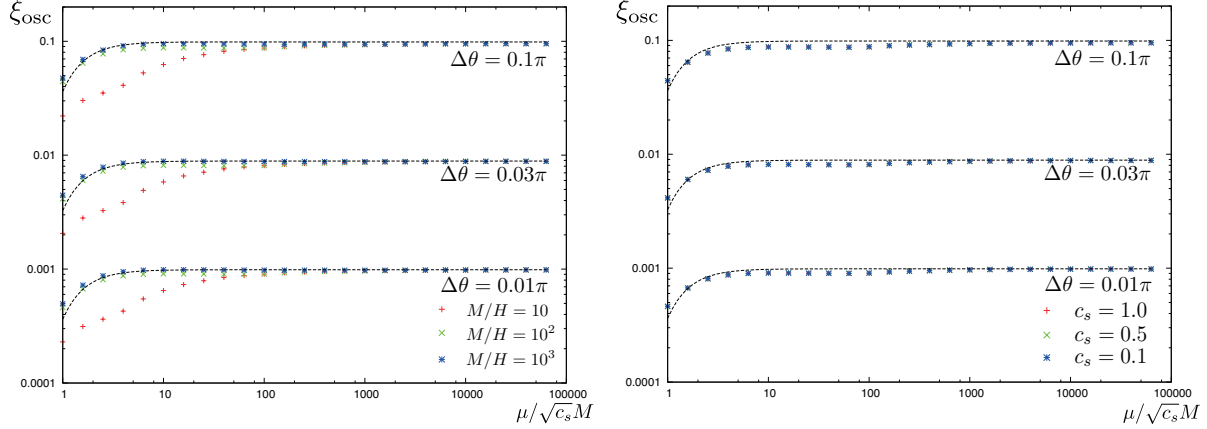


FIG. 1: Efficiency of the excitation of the heavy scalar field with respect to the sharpness of the turn μ for various values of M (left panel) and c_s (right panel). In each plot, the other parameter is fixed as $c_s = 0.5$ and $M/H = 10^2$, respectively. The points represent numerically estimated values of the efficiency, while the dotted lines correspond to the analytical estimate (38). Here, we have normalized the sharpness μ by $\sqrt{c_s}M$ since the condition for the sharp turn (40) approximately corresponds to $\mu \gg \sqrt{c_s}M$ in the present setup.

First of all, it is important to stress that this analysis uses the mass basis, which consists of the eigenvectors of the effective mass matrix for the fluctuations. This effective mass matrix contains the second derivatives of the potential but also terms that depend on the time derivatives of the scalar fields, and therefore the mass basis in general does not coincide with the potential basis defined in (15). In models with canonical kinetic terms, it turns out that the additional terms are slow-roll suppressed, so that the potential basis provides an excellent approximation of the mass basis. By contrast, in models with non canonical kinetic terms, the deviation between the mass and potential bases can become important, in particular when the sound speed is small. In this paper, we assume that the sound speed remains large enough so that these effects are not significant and can be neglected. A refined treatment, including the cases with a small sound speed, is left for future work.

Since we are interested by the final curvature power spectrum, which is ultimately observable, we concentrate on the late-time power spectrum of the light mode on super-Hubble scales, noting that the light and adiabatic direction coincide sufficiently long after the turn. The light and heavy modes being initially statistically independent, the final power spectrum of the light mode can be expressed as the sum of two contributions,

$$\mathcal{P}_{l,\text{mix}}(k) = \mathcal{P}_l^{(l)}(k) + \mathcal{P}_l^{(h)}(k). \quad (51)$$

The first contribution, $\mathcal{P}_l^{(l)}$, which we refer to as the light contribution, is obtained with initial conditions where the light mode is in its Bunch-Davies vacuum state and the heavy mode is zero. The second contribution, $\mathcal{P}_l^{(h)}$, or heavy contribution, is obtained with initial conditions where the heavy mode is in its Bunch-Davies vacuum state and the light mode is zero.

In a perturbative treatment, where $\Delta\theta$ is the small parameter, the light contribution can be formally written as

$$\mathcal{P}_l^{(l)}(k) = (1 + \mathcal{F}_l + \mathcal{F}_{lh}) \mathcal{P}_{l0} + \dots \quad (52)$$

The zeroth order term, \mathcal{P}_{l0} , is the power spectrum without any turn (i.e. $\Delta\theta = 0$). On top of it, one finds two corrections of order $\Delta\theta^2$. The first one, denoted \mathcal{F}_l , arises from the self-coupling of the light mode, proportional to $(\Delta\theta)^2$. The second correction, denoted \mathcal{F}_{lh} , comes from the coupling between the light and heavy mode, proportional to $\Delta\theta$: even though there is no heavy mode initially, this coupling generates a heavy mode with amplitude $\sim \Delta\theta$, which in turn induces a correction of order $(\Delta\theta)^2$ in the light mode.

A similar analysis can be conducted for the heavy contribution. With a non zero heavy mode initially, the coupling between the light mode and heavy mode generates a light mode of amplitude $\sim \Delta\theta$. Since the heavy contribution vanishes in the absence of mixing, this will induce a correction of order $(\Delta\theta)^2$ in the power spectrum, which we denote as \mathcal{F}_h , so that

$$\mathcal{P}_l^{(h)}(k) = \mathcal{F}_h \mathcal{P}_{l0}. \quad (53)$$

Putting everything together, the features in the power spectrum generated by the mixing are given by

$$\frac{\Delta\mathcal{P}_{l,\text{mix}}}{\mathcal{P}_{l0}} \equiv \frac{\mathcal{P}_{l,\text{mix}}}{\mathcal{P}_{l0}} - 1 = \mathcal{F}_l + \mathcal{F}_h + \mathcal{F}_{lh}, \quad (54)$$

all terms being of order $\sim (\Delta\theta)^2$. The amplitude of these corrections peaks around the scale that crosses the Hubble scale at the time of the turn t_* , i.e. $k_{\text{mix}} \equiv a_* H$ with $a_* \equiv a(t_*)$, and then decreases as k increases. In fact, as pointed out in [27], the sum of the three terms in Eq. (54) vanish in the limit $k \rightarrow \infty$.

In the case of a sharp turn, the term \mathcal{F}_h dominates on scales around k_{mix} , as \mathcal{F}_l and \mathcal{F}_{lh} tend to cancel each other on scales $H < k/a_* < m_h$, as shown in [33]. We thus need to consider only this term, whose explicit form is given by

$$\mathcal{F}_h = \lim_{k|\tau| \ll 1} \left| \int^\tau d\tau' G_l(\tau, \tau') \{ \theta_p'' u_{h0}(\tau') + 2\theta_p' u_{h0}'(\tau') \} \right|^2. \quad (55)$$

In the above expression τ denotes the conformal time and $G_l(\tau', \tau)$ is the Green's function for the light mode,

$$G_l(\tau', \tau) = 2\Im\{u_{l0}(\tau')u_{l0}^*(\tau)\}, \quad (56)$$

where u_{m0} ($m = l, h$) denote the Bunch-Davies solutions of the canonically normalized fluctuations $u_m \equiv a\delta\phi_m$, explicitly given by

$$u_{l0}(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau} \right), \quad (57)$$

$$u_{h0}(\tau) = \frac{\sqrt{\pi}}{2} e^{-\frac{\pi}{2}\nu + i\frac{\pi}{4}} \sqrt{-\tau} H_{i\nu}^{(1)}(-k\tau), \quad \nu \simeq \frac{m_h}{H}, \quad (58)$$

where $H^{(1)}$ is the Hankel function of the first kind.

It can then be shown that the maximum amplitude of the features, around the scale k_{mix} , is given by

$$\frac{\Delta\mathcal{P}_{\zeta,\text{mix}}}{\mathcal{P}_{\zeta 0}} \sim (\Delta\theta)^2 e^{-\frac{m_h^2}{\mu^2}} \left(\frac{m_h}{H} \right) \quad (59)$$

$$\sim \xi_{\text{osc}} \left(\frac{m_h}{H} \right). \quad (60)$$

In DBI inflation, the structure of the evolution equations for the perturbations is modified with respect to models with canonical kinetic terms and the features in the power spectrum are thus different in principle. However, when the sound speed is not so different from unity and when the derivative interactions remain small enough, it is natural to expect that the corrections to the features generated by the derivative interactions are not significant and that Eq. (60) remains quantitatively adequate to estimate the maximum amplitude of the features. In our specific model, we have checked numerically that this is indeed true for $c_s \geq 0.7$ with fixed m_h and μ , although we observe a shift of k_{mix} from a_*H to a_*H/c_s which can be easily explained by the modification of the sound horizon. On the other hand, for very small c_s , we observe specific effects due to derivative interactions in the features of the power spectrum, such as an enhancement of \mathcal{F}_h by a factor $\sim 1/c_s$. As mentioned previously, we will assume here that the sound speed is close enough to unity that the maximum amplitude of the features is well approximated by Eq. (60) and leave the detailed analysis of the perturbations for small sound speeds for a future work.

B. Features induced by the resonance

Since the DBI Lagrangian (3) contains derivative interactions, the inflaton fluctuations can be efficiently amplified through the resonance with the oscillations in the background trajectory [31]. When the sound speed is not so small ($f\tilde{X} \ll 1$), the square root in the Lagrangian (3) can be expanded as

$$P(X^{IJ}, \phi^I) = X_{11} + X_{22} + \frac{f}{2} [(X_{11} - X_{22})^2 + 4X_{12}^2] + \frac{f^2}{2} (X_{11} + X_{22}) [(X_{11} - X_{22})^2 + 4X_{12}^2] + \dots \quad (61)$$

Expressing the above expression in the mass basis (ϕ_l, ϕ_h) , one finds that the action contains derivative interactions between the light and heavy modes of the form

$$\frac{\lambda_{d1}}{4\Lambda_d^4} (\partial\phi_l)^2 (\partial\phi_h)^2, \quad \frac{\lambda_{d2}}{4\Lambda_d^4} (\partial\phi_l \cdot \partial\phi_h)^2, \quad (62)$$

with $\Lambda_d = f^{-1/4}$, $\lambda_{d1} = -1$, and $\lambda_{d2} = 2$, as well as the self-interaction term

$$\frac{\lambda_s}{4\Lambda_d^4} (\partial\phi_l)^4, \quad (63)$$

with $\lambda_s = 1/2$ and higher-order terms,

$$\frac{\lambda_{h1}}{8\Lambda_d^8} (\partial\phi_l)^4 (\partial\phi_h)^2, \quad \frac{\lambda_{h2}}{8\Lambda_d^8} (\partial\phi_l)^2 (\partial\phi_l \cdot \partial\phi_h)^2, \quad (64)$$

with $\lambda_{h1} = -1$ and $\lambda_{h2} = 2$, which have been considered in Ref. [31].

The feature in the power spectrum induced by the resonance has the largest amplitude around the scale that crosses the mass scale at the time of the turn, $k_{\text{res}} \equiv a_* m_h$, with

$$\frac{\Delta\mathcal{P}_{\zeta, \text{res}}}{\mathcal{P}_{\zeta 0}} \sim -\frac{1}{4} (\lambda_{d1} + 2\lambda_{d2}) q_d \sqrt{\frac{m_h}{H}}; \quad q_d \equiv f\dot{\phi}_{h,*}^2. \quad (65)$$

In the case of the DBI action, the above expression accidentally vanishes. However, if we take into account the reduction of the sound speed due to the self-interaction term (63), it can be shown that the correction has a non-vanishing contribution (see Appendix A):

$$\frac{\Delta\mathcal{P}_{\zeta,\text{res}}}{\mathcal{P}_{\zeta 0}} \sim (1 - c_s^2)q_d \sqrt{\frac{m_h}{H}}. \quad (66)$$

The parameter q_d represents the strength of the derivative interactions and can be written in terms of the sound speed (9) as,

$$q_d = f\dot{\sigma}^2 \xi_{\text{osc}} \quad (67)$$

$$= \left(\frac{1}{c_s^2} - 1 \right) \xi_{\text{osc}} \simeq (1 - c_s^2) \xi_{\text{osc}}, \quad (68)$$

where we have used $c_s \simeq 1$ in the last equality. The expression (66) was obtained by using the perturbation for the derivative interactions. The perturbation is only valid when $\Delta\mathcal{P}_{\zeta,\text{res}}/\mathcal{P}_{\zeta 0} < 1$, or

$$c_s^2 > \frac{1}{1 + (\xi_{\text{osc}} \sqrt{\frac{m_h}{H}})^{-\frac{1}{2}}}. \quad (69)$$

Here, we assume that c_s satisfies the condition (69) and that we can use the result (65) for the correction in the power spectrum due to the resonance effect. Since $\Delta\mathcal{P}_{\zeta,\text{mix}}/\mathcal{P}_{\zeta 0} < 1$ indicates $\xi_{\text{osc}} \sqrt{m_h/H} < \sqrt{H/m_h} \ll 1$, this condition is satisfied unless the features by the mixing is prominently large.

Features are also induced in the bispectrum. Introducing the dimensionless quantity \mathcal{B}_ζ defined by

$$\langle \zeta_{\mathbf{k}_1}(t) \zeta_{\mathbf{k}_2}(t) \zeta_{\mathbf{k}_3}(t) \rangle \equiv (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{P}_{\zeta 0}^2 \frac{\mathcal{B}_\zeta}{k_1^2 k_2^2 k_3^2}, \quad (70)$$

the feature reaches its largest amplitude around the scales $K \equiv k_1 + k_2 + k_3 \simeq 2k_{\text{res}}$, with

$$\Delta\mathcal{B}_{\zeta,\text{res}} \sim \max \left\{ \epsilon q_d \left(\frac{m_h}{H} \right)^{\frac{3}{2}}, (1 - c_s^2) q_d \left(\frac{m_h}{H} \right)^{\frac{5}{2}} \right\}. \quad (71)$$

Here, “max” indicates that one should take the larger term in the curly brackets because the correction to the bispectrum has contributions from both the second-order and fourth-order terms in the Lagrangian (61). In the case of the bispectrum, there is no accidental cancellation similar to that found in the power spectrum.

C. Relation between the features

Given the explicit expressions for the features in terms of the main parameters, it is now easy to find relations between them. The comparison of Eqs. (60), (66), and (71) yields

$$\frac{\Delta\mathcal{P}_{\zeta,\text{res}}}{\mathcal{P}_{\zeta 0}} \sim (1 - c_s^2)^2 \left(\frac{m_h}{H} \right)^{-\frac{1}{2}} \frac{\Delta\mathcal{P}_{\zeta,\text{mix}}}{\mathcal{P}_{\zeta 0}}, \quad (72)$$

$$\Delta\mathcal{B}_{\zeta,\text{res}} \sim \max \left\{ (1 - c_s^2) \epsilon \left(\frac{m_h}{H} \right)^{\frac{1}{2}}, (1 - c_s^2)^2 \left(\frac{m_h}{H} \right)^{\frac{3}{2}} \right\} \frac{\Delta\mathcal{P}_{\zeta,\text{mix}}}{\mathcal{P}_{\zeta 0}}. \quad (73)$$

These expressions show that the resonance cannot induce a large feature in the power spectrum, but only in the bispectrum. The relation (72) is only valid if the sound speed is not too small since we have assumed $c_s \geq 0.7$ for the mixing, as well as the condition (69) for the resonance. The latter condition is automatically satisfied if the resonance feature has a reasonable amplitude, $\Delta\mathcal{P}_{\zeta,\text{res}}/\mathcal{P}_{\zeta 0} < 1$. Thus, the condition $c_s \geq 0.7$ is sufficient to ensure the validity of the relation (72) as far as we consider reasonable modulations in the power spectrum. This condition is not so restrictive to get a large modulation in the bispectrum since the factor m_h/H can be sufficiently large to compensate the suppression by the factor $(1 - c_s^2) \leq 0.5$ for $c_s \geq 0.7$.

Note that the amplitude (73) is written only in terms of observable parameters. The sound speed and the slow-variation parameter can be deduced in principle from the observations of the tensor modes and equilateral non-Gaussianity: $r = 16c_s\epsilon$, $n_t = -2\epsilon$, and $f_{\text{NL}} \sim -(35/108)(c_s^{-2} - 1)$. Moreover, m_h/H can be determined by the relative position of the two features, according to

$$\frac{m_h}{H} = \frac{k_{\text{res}}}{k_{\text{mix}}}. \quad (74)$$

Therefore, given a mixing feature observed in the power spectrum, one could look for a corresponding resonance feature in the bispectrum. The observation of such combination of features in the data would provide a strong evidence for the presence of a heavy mode, excited during inflation.

V. SUMMARY

In this paper, we have analysed a sharp bending of the trajectory in two-field DBI inflation, where two types of features in the primordial spectra are expected to be induced, due to mixing and resonance effects, respectively. Indeed, in addition to features induced by a large mixing between the light and heavy modes, derivative interactions, which are contained in the non-canonical kinetic term of the DBI action, can lead to an efficient enhancement of the fluctuations through a resonance with the oscillations of the heavy field excited by a sharp turn.

To see the relation between these two features, we have investigated how the amplitude of the oscillations is determined by the details of the turn, taking into account the effect of the derivative interactions, i.e. the reduction of the sound speed. Introducing the efficiency parameter, defined as the ratio between the maximum kinetic energy of the heavy field and the total kinetic energy before the turn, we found that it is determined only by the global variation of the light direction angle in the sharp-turn limit.

With the assumption that the sound speed remains rather large ($c_s \geq 0.7$), we find that the maximum amplitude of the both mixing and resonance features does not depend on the information of the turn other than the efficiency parameter. Remarkably, it is then possible to derive consistency relations between the two types of features. As consequences, it was shown that the resonance cannot induce a large feature in the power spectrum, but only in the bispectrum due to a larger amplification factor of the heavy mass. The resultant resonance features are suppressed by a factor of $1 - c_s^2$, which represents the strength of the derivative interactions. However, even when the kinetic term can be approximated to be canonical, $c_s \simeq 1$, the amplification factor is large enough to realize a non-negligible feature in the bispectrum when the heavy mass is much larger than the Hubble scale.

If the combination of both mixing and resonance features were detected and shown to satisfy the consistency relation, it would give a compelling evidence for the existence of a heavy field during inflation. Moreover, the relative position of the features would give directly the value of the heavy mass. It would thus be interesting to look for such features in the data, following techniques that have been developed in e.g. Refs. [42–51].

In this paper, we have not discussed the cases with a small sound speed $c_s < 0.7$. None of the current data point to a small sound speed, but there is still room for it, since the current constraints on equilateral non-Gaussianity give the lower bound $c_s > 0.07$ at 95% CL [52] (assuming a single light field). As already mentioned, the results of the present work do not apply for a small sound speed and further investigations are required. We will study these cases in a future work.

Acknowledgments

D.L. was partly supported by ANR (Agence Nationale de la Recherche) grant “STR-COSMO” ANR-09-BLAN-0157-01. S.M. is grateful to the APC for their hospitality when this work was almost done. R.S. is supported by Grant-in-Aid for JSPS postdoctoral fellowships for research abroad.

Appendix A: Resonance features in the power spectrum for the DBI action

In the case of DBI, it turns out that the correction to the power spectrum due to the resonance, which has been obtained by a perturbative expansion with respect to the derivative interactions, accidentally vanishes. In this appendix, we show that the correction is nevertheless non-vanishing if one takes into account the reduction of the sound speed due to the self-interaction term (63) included in the DBI action.

To see the general structure of the correction, we first calculate it without fixing the values of λ_{d1} , λ_{d2} , and λ_s , keeping in mind that

$$\lambda_{d1} = -1, \quad \lambda_{d2} = 2, \quad \lambda_s = \frac{1}{2}, \quad (\text{A1})$$

for the DBI action. For brevity, we introduce the quantity that controls the amplitude of the self-interaction term,

$$q_s \equiv f \dot{\phi}_t^2. \quad (\text{A2})$$

Comparing the definition of q_d , which controls the amplitude of the interaction between the light and heavy modes, we find

$$\frac{q_d}{q_s} = \frac{\xi_{\text{osc}}}{1 - \xi_{\text{osc}}}. \quad (\text{A3})$$

Hence, though q_s is slow-roll suppressed, it has a value larger than q_d . It contributes to the non-oscillatory part of the sound speed as

$$\bar{c}_s^2 = \frac{1 + \lambda_s q_s}{1 + 3\lambda_s q_s}. \quad (\text{A4})$$

When the resonance is not relevant, it gives the leading contribution to the sound speed.

The quadratic Hamiltonian in this system is given by [40, 41],

$$H^{(2)} = \int d^3x \frac{1}{2} \left[\dot{v}^2 + \left(\frac{c_s^2 \nabla^2}{a^2} - \frac{\ddot{z}_\phi}{z_\phi} \right) v^2 \right]. \quad (\text{A5})$$

where

$$z_\phi^2 \equiv a^3 (2P_{1K,L1} X^{KL} + P_{11}), \quad (\text{A6})$$

$$c_s^2 \equiv \frac{P_{11}}{2P_{1K,L1} X^{KL} + P_{11}}, \quad (\text{A7})$$

and $v \equiv z_\phi \delta\phi_l$. Extracting the leading-order oscillatory components, we find

$$z_\phi^2 = a^3 \left[1 + 3\lambda_s q_s + (\lambda_{d1} + \lambda_{d2}) q_d \sin^2(m_h t) \right], \quad (\text{A8})$$

$$c_s^2 = \bar{c}_s^2 \left[1 + \frac{1}{1 + 3\lambda_s q_s} \left(\frac{\lambda_{d1}}{\bar{c}_s^2} - \lambda_{d1} - \lambda_{d2} \right) q_d \sin^2(m_h t) \right] + \mathcal{O}(q_d^2). \quad (\text{A9})$$

Here, we did not perform an expansion with respect to q_s unlike the treatment in Ref. [31] to see the effect of the reduction in the sound speed due to the self-interaction, $\bar{c}_s^2 < 1$.

Using the method of steepest descent, it can be shown that the peak amplitude of the correction to the power spectrum is given by

$$\frac{\Delta \mathcal{P}_{\zeta, \text{res}}}{\mathcal{P}_{\zeta 0}} \sim \left(\frac{c_s^2}{\bar{c}_s^2} - \frac{\ddot{z}_\phi}{z_\phi} \right) \Big|_{\text{osc}} \sqrt{\frac{m_h}{H}}, \quad (\text{A10})$$

where “osc” in the subscript indicates that one should pick up the oscillatory components. From Eqs. (A8) and (A9), it can be estimated as

$$\left(\frac{c_s^2}{\bar{c}_s^2} - \frac{\ddot{z}_\phi}{z_\phi} \right) \Big|_{\text{osc}} \simeq -\frac{q_d}{2(1 + 3\lambda_s q_s)} \left(\frac{\lambda_{d1}}{\bar{c}_s^2} + \lambda_{d1} + \lambda_{d2} \right), \quad (\text{A11})$$

where we have discarded terms suppressed by H/m_h . Then, substituting the DBI values given in (A1),

$$\left(\frac{c_s^2}{\bar{c}_s^2} - \frac{\ddot{z}_\phi}{z_\phi} \right) \Big|_{\text{osc}} \simeq \frac{q_d}{2(1 + 3\lambda_s q_s)} \left(\frac{1}{\bar{c}_s^2} - 1 \right), \quad (\text{A12})$$

$$\simeq \frac{q_d}{2} (1 - \bar{c}_s^2), \quad (\text{A13})$$

where we have used $q_s \ll 1$ and then $\bar{c}_s^2 \simeq 1$ in the last equality. Therefore, although the resonant feature vanishes at the leading order of the derivative interactions in the DBI case, it is no longer the case if one takes into account the reduction of the speed sound due to the self-interaction term.

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